Developmental Mathematics: A Plan for Florida's Community Colleges

Norma M. Agras, Miami-Dade College nagras@mdc.edu
Project Director
Retreat Coordinator
Discussion Facilitator, Curriculum Issues, Retreat 2002

Martha Goshaw, Seminole Community College GoshawM@scc-fl.edu
President, Florida Two Year College Mathematics Association
Discussion Facilitator, Teaching Strategies, Retreat 2002

Cliff Morris, Jr., Valencia Community College cmorris@valencia.cc.fl.us
Past President, Florida Two Year College Mathematics Association
Discussion Facilitator, Testing, Assessment, and Student Support, Retreat 2002

Curriculum Committee
John Salak, Tallahassee Community College, Committee Chair Tccmathguy@aol.com
Byron Dyce, Santa Fe Community College byron.dyce@santafe.cc.fl.us
Andrew Lambert, Palm Beach Community College lamberal@pbcc.edu
Don Ransford, Edison Community College dransford@edison.edu

A Project of the Florida Two Year College Mathematics Association

This plan was developed from discussions among, and input from, faculty from the following school districts, colleges and universities:

Brevard Community College
Broward Community College
Central Florida Community College
Daytona Beach Community College
Eckerd College
Edison Community College
Embry Riddle Aeronautical University
Daytona Beach Community College
Florida College
Florida Community College at Jacksonville
Florida Gulf Coast University
Gulf Coast Community College
Hillsborough Community College
Hillsborough Public Schools
Indian River Community College
Jacksonville University
Lake City Community College
Lake-Sumter Community College
Manatee Community College
Miami-Dade Community College
North Florida Community College
Okaloosa-Walton Community College
Palm Beach Community College
Pasco-Hernando Community College
Pensacola Junior College
Pinellas County Schools
Polk Community College
Portland Community College, Oregon
Saint Johns River Community College
Saint Petersburg College
Santa Fe Community College
Seminole Community College
Tallahassee Community College
United States Military Academy, West Point
University of North Carolina, Chapel Hill
University of South Florida
Valencia Community College
Washington University
Wentworth Institute of Technology

Foreword

In March 2002, the Joint Meetings of the Florida Section of the Mathematical Association of America (MAA) and the Florida Two Year College Mathematics Association (FTYCMA) were held at Santa Fe Community College in Gainesville, Florida. At a well-attended presentation by Norma Agras, "Mathematics in Florida: Technology-Propelled Changes," instructors from around the state were asked to look critically at the state's existing curriculum.

> *"The curriculum in pre-college-level courses seems to ignore the needs of today's students both in content and in pedagogy…We need to determine what we expect students to know – and know well – prior to enrolling in college-level mathematics courses, both calculus-track and non-calculus track, and then as a consequence of having completed such courses. From such a study, we can begin to build a foundation that will better prepare students for mathematics-dependent disciplines as well as provide our state with college graduates who are quantitatively literate." (Norma Agras)*

Those who attended this session were enthusiastically in favor of having a State Retreat at which faculty around the state could gather to begin to address these issues. At the FTYCMA business meeting held later that year, there was unanimous support of having such a Retreat, to be named Mathematics in the Sun, and thus the first Retreat was scheduled for fall.

In September 2002, mathematics instructors from around the state of Florida gathered at the Tarpon Springs campus of St. Petersburg College for the first annual Retreat. Sponsored by FTYCMA, the Retreat provided these educators with the opportunity to share concerns and ideas with regard to various facets of teaching developmental mathematics. Developmental mathematics was defined as all mathematics courses below the level of College Algebra. This document consists of recommendations that were made at the first Retreat, input from discussion at two subsequent workshops, and suggestions received from around the state over the next several months, with mathematics educators having met again in fall 2003 for the second Retreat.

The recommendations made by this Plan are consistent with, and model, the Standards of the American Mathematical Association of Two Year Colleges (AMATYC).

Part I. Project Goals

- To improve the quality of developmental mathematics programs in Florida and increase success and retention rates in those programs
- To improve the success rate in college mathematics courses of students who successfully complete developmental mathematics programs
- To improve the mathematics preparation of developmental students who take quantitative courses in disciplines other than mathematics

Part II. Recommendations for Curriculum/Courses

Recommendations in A and B (below) are based upon discussions at Retreat 2002 and on subsequent input received until the creation of the Curriculum Committee in February, 2003. Further, it was universally agreed at Retreat 2002 that developmental mathematics is a foundation for college mathematics, not a rapid-paced repetition of high school algebra.

A. Course/program guiding principles

Number sense: Students should be able to estimate, and to determine the reasonableness of a result. Students should be able to perform basic calculations without the use of a calculator. Students should be able to use the proper unit of measure, and convert within a system of measure. Precision should be expected.

Geometry: Students should have learning experiences that provide an opportunity for conceptual understanding of topics of informal geometry, including measurement (perimeter, area, volume), similarity, and basic properties and classifications of angles and polygons.

Communication: Students should be able to read a problem, determine what is known and what needs to be computed or found, rewrite the problem in symbolic form, solve it, and justify the results. Students should be able to use proper mathematical language to ask questions and to describe the process by which they arrive at a result.

Multiple representations: Students should be able to understand the connection among verbal, symbolic, numerical, and graphical representations. Students should be able to represent contextual problems symbolically and graphically.

B. Course/program curriculum

Students completing the developmental curriculum should be able to solve problems that, within their solutions, could require that a student:

Evaluate or simplify expressions
Translate verbal expressions into symbolic form

Example: (COMAP, Developing Mathematics through Applications, Key College Press) Assume brewed tea has 45 mg of caffeine per 8-oz cup, soft drinks containing caffeine have 55 mg per 12 oz can, and chocolate contains 10 mg of caffeine per bar. Calculate the caffeine consumed by a person having one 8-oz cup of tea, two 12-oz cans of soft drinks, and three bars of chocolate. Suppose a person consumes *s* 12-oz cans of sodas, *b* bars of chocolate, and *t* 8-oz cups of tea. Write an expression for the amount of caffeine consumed. Find a combination of tea, soda and chocolate that results in 175 mg of caffeine.

Solve first-degree equations

Plot and interpret data, create models, and make reasonable predictions from the models

Read and interpret a graph

Write equations for quantities that vary directly or inversely to each other

Graph linear equations

Example: On the same coordinate system, draw the graphs of three lines that pass through the point (2,1). Write an equation for each.

Interpret slope in applications

Example: (COMAP, Developing Mathematics through Applications, Key College Press) A gas company charges its residential customers a base rate of \$32. In addition, for each therm of gas used, the company charges \$0.64. Write an equation that relates the total monthly bill B (in dollars) to the amount of gas g (in therms.) Then determine how much gas was used if the bill for one month totals \$97.23.

Example: (COMAP, Developing Mathematics through Applications, Key College Press) According to Baby Bag online, the average weight for an 8-year-old girl is $60\frac{3}{4}$ pounds. The average weight for a 12-year-old girl is 94 pounds. Use this information to create a linear model. Let *w* represent weight and *a* represent age. Express your model in slope-intercept form, y=mx+b. What does the line mean in context? Does the y-intercept have any meaning in context? Explain. Use your model to predict the average weight of a 9-year-old girl.

Solve and graph systems of linear equations and interpret the solution

Example: For what value of x does the expression $y_1 = 3x + 1$ have the same value as $y_2 = -x + 5$? Graph both equations and locate the point of intersection on the graph.

Solve inequalities

Graph inequalities in two variables

Evaluate expressions that contain exponents

Example: The value of the expression 2^x increases as x increases. Evaluate (mentally) 2^3 and 2^4, then estimate $2^{3.1}$ and $2^{3.9}$. Check your answers with a calculator.

Write very small or very large numbers in scientific notation

Example: (COMAP, Developing Mathematics through Applications, Key College Press) The world's population reached 6 billion in October 1999 and was then growing at the rate of 78 million per year. Express each of these numbers in scientific notation. If the rate of world population growth stayed the same, what would be the population of the world in October 2010?

Know basic formulas of Euclidean geometry, including Pythagorean Theorem, area, volume, and be able to apply these to solve problems

Example: (COMAP, Developing Mathematics through Applications, Key College Press) As part of the "Big Dig" harbor reconstruction project in Boston, a casting basin had to be created to allow construction of new underwater tunnel connections. The average dimensions of the concrete canyon were 50 ft deep, 250 ft wide, and 1000 ft long. If we model the basin as a rectangular solid, what is the name of the quantity you would compute in order to determine how much ground space was taken up by the casting basin? Find its value using the given units.

Example: Find the area of a rectangular region with length 10 meters and diagonal 16 meters.

Example: (COMAP, Developing Mathematics through Applications, Key College Press) Write a polynomial expression to represent the volume of a cylinder with diameter *n* and height *2n*. Write a polynomial to represent the surface area of the cylinder.

Solve literal equations or formulas for a specified variable, especially formulas from the natural or social sciences

Example

Solve for the specified variable

$$V = \frac{4}{3}\pi r^3, \quad for\ r$$

$$A = A_0(1+r)^n, \quad for\ r$$

Solve problems involving proportions, especially similarity problems of Euclidean geometry

Graph linear and simple quadratic functions

Example Draw the graph of $y_1 = 2x \quad and \quad y_2 = x^2$ on the same coordinate system. Determine the coordinates of the points where the graphs intersect.

Determine if a given relation is a function, especially if it is given as a table of values or a graph

Use function notation

Solve quadratic equations and identify equations that have imaginary solutions

Perform basic operations on polynomials

Example: Find the quotient of $y^2 - 1$ *and* $y - 1$. Check by multiplying.

Example: Square $a - 4$

Factor polynomials (decreased emphasis)

Perform operations on simple algebraic fractions (decreased emphasis; possible optional topic)

Examples

Find the sum of $\frac{1}{x^2 y} \quad and \quad \frac{2}{xy^3}$

Find the product of $\frac{x+1}{x^2-4}$ *and* $\frac{x-2}{x^3}$

Find the quotient: $\frac{x-1}{6x^2y} \div \frac{3x-3}{2xy^4}$

C. Courses

The preference at this time is for the developmental curriculum to consist of two courses. Some colleges will also continue to offer MAT 0002 or some version of arithmetic, as necessary, as a possible third course. Additionally, it is desirable that some colleges pilot alternative approaches, such as a modular approach or a self-guided approach with an instructor present.

Another possibility for courses includes a two-semester sequence that would combine the material of this “bridge” course with college algebra in such a way that students learn (or review) the material of the “bridge” course as they need it for the college algebra topics.

A Curriculum Committee was created at the February, 2003, Joint Meeting of FTYCMA and the Florida Section of the MAA to design a new course that would serve as the pre-requisite to College Algebra and to Mathematics for Liberal Arts. Unlike the existing MAT 1033 (Intermediate Algebra) course that is often skills-based, the newly designed course is to be concept-based, rich in contextual problems and real world applications that would serve as a platform from which to learn these concepts. This new course is based on the curriculum given in part B of this section, with the guiding principles of Part A of this section. The Curriculum Committee consists of Byron Dyce, Santa Fe Community College, Andrew Lambert, Palm Beach Community College, Don Ransford, Edison Community College, and John Salak, Committee Chairman, Tallahassee Community College.

D. Content of the redesigned pre-college level mathematics course: Recommendations of the Curriculum Committee

> *The following list contains the ideals which might be included as part of our ‘bridge course’. The committee would like to thank our colleagues from around the state who helped us refine our original brainstorming. We have received e-mails, written notes, and of course had lengthy sessions collaboratively to arrive at this list. Thank you so much for your contributions. As mathematics educators, let’s work together as a group to see this ‘bridge course’ become reality. If we simply let this list be on paper and do not fight for its development, we have not helped our students.*
>
> *Instructors of this course should reinforce concepts from previous course as frequently as necessary to facilitate learning. Our actual ‘bridge course’ will contain a proper subset of these ideals listed (below). Also, some recurrent themes from educators around the state should be mentioned at the closure of this list. This course should emphasize quality and not quantity. We would prefer fewer concepts in depth than many concepts superficially. We also need time to*

teach this course, at least four contact hours! (John Salak, on behalf of the Curriculum Committee)

The following four recommendations of the Curriculum Committee are consistent with those found in part A of this section. These are not intended to be course competencies but rather they are the fundamental principles that guide the content or curriculum and the learning opportunities throughout the entire course.

Recommendation 1. This course should empower students to know the vocabulary of mathematics and use it effectively.

- Recognize, differentiate among, and define algebraic terminology such as constant, variable, term, factor, polynomial, exponent, base, expression, equation;
- translate from verbal to symbolic form and conversely;
- recognize order relations in verbal and symbolic form such as at least, at most, greater than, no more than, etc.;
- understand the meaning of solve, solution, solution set, no solution

Recommendation 2. This course should empower students to demonstrate quantitative literacy and number sense.

- Give an answer in the context of a problem, using appropriate units of measure to any desired accuracy;
- solve problems involving proportions and percents;
- understand significant digits and order of magnitude;
- express very large or very small quantities in scientific notation;
- recognize and name geometric shapes;
- state, recognize and use geometric formulas to compute measures of angles, lengths, areas, volumes, and use appropriate units of measure;
- think critically and solve problems involving the concepts of reasonableness and estimation to improve number sense

Recommendation 3. This course should empower students to think analytically and solve problems.

- Solve challenging problems that require analytical reasoning;
- solve fun problems and puzzles that improve problem solving ability;
- learn to think logically by regularly solving a variety of problems, especially real world applications;
- write out in English phrases the method to approach and solve a problem

Recommendation 4. This course should empower students to use technology effectively and appropriately.

- Use technology effectively and appropriately, particularly hand-held calculators, for graphing, problem solving, and performing tedious calculations;
- gain computer proficiency with the goal of utilizing resources such as using web pages and the Internet to reinforce mathematics concepts throughout the course

Some of the concepts that would be studied in this course include the following.

Students will explore relations between variables in multiple formats (numerical, graphical, algebraic, verbal); in particular, students will

- read and interpret graphs;
- graph linear and basic quadratic functions;
- understand the geometric significance of slope and interpret slope in context;
- make the connection between functional notation $y = f(x)$ and an ordered pair (x,y) on the graph of a function;
- understand the connection between the roots of the equation $f(x) = 0$, the zeros of the function $y = f(x)$, and the x-intercepts of the graph of the function $y = f(x)$;
- recognize causal relationships between quantities given graphically, numerically, verbally, symbolically;
- see relationships between variables;
- identify the intervals over which a function, given its graph, is increasing, decreasing, or constant;
- understand the concept of domain and determine the most appropriate domain for a given function;
- understand the concept of a mathematical model;
- given a set of data, find the linear model that best fits the data and interpolate or extrapolate to make estimates or predictions

Students will simplify expressions, and will solve equations and inequalities; in particular, students will

- recognize and apply the Laws of Exponents;
- use radical notation or rational exponent form as appropriate;
- understand the definition of a logarithm and convert between logarithmic and exponent form;
- perform simple logarithmic computations;
- classify equations as linear, quadratic, exponential, etc.;
- use laws of equality and laws of inequality to solve equations and inequalities;
- use formulas from a variety of contextual applications including business, industry, social science, and the natural sciences;
- solve a variety of literal equations for a specified variable;
- understand the concept of number systems in conjunction with equation solving;
- discuss forms for solutions and solution sets and determine which form might best suit a particular type of equation or inequality

Part III. Recommendations for Teaching Strategies

Developmental mathematics programs must be activity based rather than solely instructor led. Instructors must meet students at their existing level of mathematics knowledge and understanding, and incorporate strategies and techniques that facilitate students' learning and progression to the next level. The techniques that might work in a high school setting are often not successful with college students, especially adult learners. The ideal developmental mathematics class should be activity-oriented, include group work, and use technology as a tool to enhance instruction.

A. Recommended Teaching Strategies

Recommendation 1: Collaborative/cooperative strategies should be implemented as a method of enhancing and facilitating learning. Activities could include:

- Group quizzes and projects
- Group work to develop a topic
- Group board work
- Review of material

Recommendation 2: Technology, including calculators and computers as available, should be used as appropriate as a tool for learning. Note: Students should be encouraged to perform simple arithmetic calculations mentally.

Calculators could be used for:

- Reinforcement of estimation skills and reasonableness of answers
- Observation of relationships between two quantities

Computers could be used for:

- Mediated learning
- PowerPoint demonstrations
- Communication (email)
- Instructor web pages
- Course web pages
- Links to mathematics sites
- Resource of information for projects
- Spreadsheet programs

Recommendation 3: Developmental mathematics programs must focus on communication skills (reading, writing, speaking). Note: The use of proper mathematical language should be stressed. Activities could include:

- Having students read a mathematics book, at home or aloud in class, including their course's designated textbook or a supplemental textbook.

- Including questions on tests and quizzes that require a student to write and explain a procedure, such as how to solve a given problem.
- Assigning oral and written projects

Recommendation 4: Because reading is an integral part of a mathematics course, a reading level requirement should be instituted as an entry requirement for developmental mathematics classes.

B. Successful Strategies

Listed below are some strategies that have been implemented successfully by various project participants with developmental mathematics students. Developmental mathematics programs should incorporate as many of these as possible.

- Student-centered instruction, cooperative learning and group work
- Appropriate use of technology, both calculator and computer
- Journals and writing exercises
- Instructions on how to use solutions manuals and other resources
- Extra help sessions, especially before tests
- Solution of the same problem from different viewpoints.
- Assistance with improvement of mathematics study skills; for example, having students keep a homework notebook and record any questions or problems in the margins
- Use of personal web pages as a way of increasing student access to course information
- Positive feedback and recognition of successes, especially for adult learners.
- Learning communities within the classroom through facilitation of creation of study groups in class at the beginning of the term
- Providing clear course structure
- Pairing developmental mathematics classes with SLS or college survival skills-type classes
- Exercises and problem-solving opportunities that allow students to work at the concrete level to a level of conceptual understanding and abstraction

C. Instructor Training and Communication

A developmental mathematics program is only as successful as the quality and the level of involvement of its instructors. Since adjunct instructors teach a large proportion of the developmental mathematics classes that are offered by our institutions, these instructors are a vital part of the program's success.

Recommendation 1: Pair developmental mathematics adjuncts with full time developmental mathematics faculty for mentoring and informal evaluation.

Recommendation 2: Place a packet of information on the college's web site for easy access by adjuncts. This packet could include contact information for full time faculty and the department head, sample syllabi and tests, policy and procedure information, and any other items deemed important by the college or department.

Recommendation 3: Train full time and adjunct instructors in the appropriate use of technology, in collaborative learning, in student-centered instruction, in the basics of how to communicate with students in developmental programs, in appropriate assessment of student learning.

Part IV. Recommendations for Testing and Assessment

Most participants of two-year colleges represented acknowledged little or no connection between the FCAT (Florida Comprehensive Assessment Test) which measures critical thinking skills and/or concepts achieved in high school and the CPT (Computerized Placement Test), which is a mathematical skills placement test, administered to registering community college students. Participants were confident, though not universally so, that the CPT is doing a good job with initial placement of entering students into the existing courses. Redesigning the developmental program will likely require the rethinking of placement, with a high probability of changes to the existing procedure(s).

Recommendation 1: For refining placement beyond the CPT for students who place into at least the current MAT1033, the following are currently in use and are recommended:

- Looking at high school transcripts
- Interviewing of students by mathematics faculty
- Further testing by the academic department
- Using the CLM (College Level Mathematics, a CPT subtest)

Recommendation 2: Multiple forms of assessment should be used to evaluate student comprehension. Appropriate assessment strategies that could be used in addition to exams:

- Homework
- Computer assignments
- Cooperative learning activities
- Quizzes to assess progress
- Group quizzes to foster individual and peer responsibility among students
- Projects, oral and written, based on real world themes
- Portfolios to document individual progress by and for each student
- Writing assignments
- Laboratory-based activities in stations similar to those used for science classes

The use of multiple choice questions on exams should be very limited, with various educators recommending their use only on final exams, and the items should be well-written to test conceptual understanding, definitions, and interpretation of results. Technology may be used on exams depending on what is being tested. One group stated, "Students are more able to demonstrate understanding in different ways and should also be expected to articulate their understanding in different contexts. It is critical that students can show <u>how</u> they can use this knowledge."

<u>Recommendation 3</u>: The College Preparatory Exit Test (CPET) was almost unanimously considered to serve no useful purpose. Most participants preferred using a departmental final exam, or at least standardization of regulations and statewide minimum passing scores on the CPET.

Part V. Recommendations for Physical Setting and Institutional Support

The ideal developmental mathematics classroom has a physical setting that is conducive to group work. The developmental classroom setting must support and enhance instruction and learning, and should facilitate the fostering of a sense of community among the students.

<u>Recommendation 1</u>: Class size must be appropriate. Maximum class size recommendation by the American Mathematical Association of Two Year Colleges (AMATYC) is 30 students per class.

<u>Recommendation 2</u>: Four to five weekly contact hours are necessary to allow the instructor to teach the necessary material in a meaningful way.

<u>Recommendation 3</u>: The time frame of the class must be conducive to the various activities that take place; 50-minute time blocks are too short.

<u>Recommendation 4</u>: The arrangement of the classroom must support the instructional strategies. In particular,

- Ample black board space is imperative.
- Computer screens should not block board space.
- Tables should be provided for group activities.
- Floor space is needed for the instructor to circulate among the students as they work.

Part VI. Implementation and Future Discussion of the Plan

Those receiving this document should proceed to share it with their colleagues for discussion and to begin discussion for local implementation needs. Assessment strategies, sample teaching strategies modeling this Plan, and implementation discussions will take place at the Joint Meetings of FTYCMA and MAA Florida Section, at Manatee Community College, Florida, in February 2005. Please look at the web site FTYCMA.org for updates.